

Joule heating in spin Hall geometry

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The theoretical formula for the entropy production rate in the presence of spin current is derived using the spin-dependent transport equation and thermodynamics. This theory is applicable regardless of the source of the spin current, for example, an electric field, a temperature gradient, or the Hall effect. It reproduces the result in a previous work on the dissipation formula when the relaxation time approximation is applied to the spin relaxation rate. By using the developed theory, it is found that the dissipation in spin Hall geometry has a contribution proportional to the square of the spin Hall angle.

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Reducing the power consumption in spintronics devices is an important problem for both fundamental physics and practical applications. Therefore, the development of a theory of dissipation (heating) due to the spin current is an attractive topic in this field¹⁻⁴. In 2011, Tulapurkar and Suzuki¹ investigated the dissipation in a one-dimensional ferromagnetic multilayer using the Boltzmann equation, where the spin current is driven by the electric field E_x . In this case, the electric current density carried by spin- ν electrons ($\nu = \uparrow, \downarrow$ or \pm) is expressed as $J_{c,\nu} = \sigma_\nu E_x$, where σ_ν is the conductivity of the spin- ν electrons. They found an additional contribution proportional to $\beta^2/(1 - \beta^2)$ to the conventional Joule heating σE_x^2 , where $\sigma = \sigma_\uparrow + \sigma_\downarrow$ and $\beta = (\sigma_\uparrow - \sigma_\downarrow)/\sigma$ are the total conductivity and its spin polarization, respectively.

On the other hand, several methods of generating spin current, such as nonlocal spin-injection by diffusion⁵⁻⁷, spin pumping by ferromagnetic resonance⁸⁻¹⁰, the spin Hall effect due to spin-orbit interaction¹¹⁻¹⁶, the spin Seebeck effect due to heating¹⁷⁻²¹, and spin hydrodynamic generation by fluid dynamics^{22,23}, have recently been proposed theoretically and also demonstrated experimentally. The spin currents driven by these effects are not described by the Boltzmann equation used in Ref.¹. From this perspective, it is unclear whether the dissipation formula derived in a previous work¹ is still applicable to these cases. Therefore, it is desirable to derive a dissipation formula of the spin current, that is independent of the explicit form of the source term (driving force) of the current.

In this letter, we derive the theoretical formula of the entropy production rate for systems having spin-dependent transport properties by using the continuous equation of the spin current and thermodynamics. The derived formula is applicable regardless of the source of the spin current because it requires no explicit specification of the form of the current. We also show that the present formula reproduces the dissipation formula derived by Tulapurkar and Suzuki¹ when the relaxation time approximation is applied to the spin relaxation rate. We apply the present formula to derive the dissipation in the spin Hall geometry, and find a contribution propor-

tional to the square of the spin Hall angle.

We first consider the spin-dependent transport of electrons in condensed matter, where the particle density n_ν and current density \mathbf{j}_ν of spin- ν electrons satisfy

$$\frac{\partial n_\nu}{\partial t} + \nabla \cdot \mathbf{j}_\nu = -\varphi_\nu, \quad (1)$$

where φ_ν is the spin relaxation rate. Because of the conservation law of the particle number, φ_ν should satisfy $\varphi_\uparrow + \varphi_\downarrow = 0$. The particle current density \mathbf{j}_ν is related to the electric current density via $\mathbf{J}_{c,\nu} = -e\mathbf{j}_\nu$, where $e = |e|$ is the elementary charge. The total electric current density is $\mathbf{J}_c = \mathbf{J}_{c,\uparrow} + \mathbf{J}_{c,\downarrow}$, whereas the spin current density is $\mathbf{J}_s = [\hbar/(-2e)](\mathbf{J}_{c,\uparrow} - \mathbf{J}_{c,\downarrow})$. There are several sources of the current \mathbf{J}_c (or $\mathbf{J}_{c,\nu}$). For example, when an external electric field \mathbf{E} drives the current, \mathbf{J}_c is given by $\sigma\mathbf{E}$. On the other hand, the current density is given by $\mathbf{J}_c = \sigma S \nabla T$ when a temperature gradient drives the current, where S and T are the Seebeck coefficient and the temperature, respectively. We emphasize that the following derivation of the entropy production rate formula is independent of the explicit form of \mathbf{J}_c . In other words, the following calculation is applicable not only to the one-dimensional system with an electric field studied in Ref.¹ but also to other systems.

The total energy density \mathcal{E} consists of the internal energy density u and potential energy density W , where $W = -\sum_{\nu=\uparrow,\downarrow} en_\nu V$ with an electric voltage V . Therefore, the change in the total energy density \mathcal{E} is

$$\frac{\partial \mathcal{E}}{\partial t} = \frac{\partial u}{\partial t} + \frac{\partial W}{\partial t}. \quad (2)$$

Similarly, the energy current density \mathbf{j}_E is related to the internal energy current density \mathbf{j}_u and the particle current density of the spin- ν electrons \mathbf{j}_ν via

$$\mathbf{j}_E = \mathbf{j}_u + \sum_{\nu=\uparrow,\downarrow} (-eV)\mathbf{j}_\nu. \quad (3)$$

The total energy density \mathcal{E} and the energy current density \mathbf{j}_E satisfy the energy conservation law

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \mathbf{j}_E = 0. \quad (4)$$

In the steady state, $\nabla \cdot \mathbf{j}_E = 0$. Then, we find that the Joule heating formula, $\nabla \cdot \mathbf{j}_u = \mathbf{J}_c \cdot \mathbf{E}$, is reproduced from Eq. (3) and the conservation law of the particle number, $\sum_{\nu=\uparrow,\downarrow} \nabla \cdot \mathbf{j}_\nu = 0$, where the electric field is related to the electric voltage V via $\mathbf{E} = -\nabla V$.

According to thermodynamics, the change in the internal energy density is²⁴

$$\frac{\partial u}{\partial t} = T \frac{\partial \mathfrak{S}}{\partial t} + \sum_{\nu=\uparrow,\downarrow} \mu_\nu \frac{\partial n_\nu}{\partial t}, \quad (5)$$

where \mathfrak{S} is the entropy density, and μ_ν is the chemical potential of the spin- ν electrons. The electrochemical potential is $\bar{\mu}_\nu = \mu_\nu - eV$. We also define the heat current density \mathbf{j}_q as

$$\mathbf{j}_q = \mathbf{j}_u - \sum_{\nu=\uparrow,\downarrow} \mu_\nu \mathbf{j}_\nu. \quad (6)$$

From Eq. (3), the energy and heat current densities are related as

$$\mathbf{j}_E = \mathbf{j}_q + \sum_{\nu=\uparrow,\downarrow} \bar{\mu}_\nu \mathbf{j}_\nu. \quad (7)$$

Using Eqs. (1)-(7), we find that

$$T \frac{\partial \mathfrak{S}}{\partial t} = -\nabla \cdot \mathbf{j}_q - \sum_{\nu=\uparrow,\downarrow} \mathbf{j}_\nu \cdot \nabla \bar{\mu}_\nu + \sum_{\nu=\uparrow,\downarrow} \bar{\mu}_\nu \varphi_\nu. \quad (8)$$

The equation can be rewritten as

$$\frac{\partial \mathfrak{S}}{\partial t} + \nabla \cdot \frac{\mathbf{j}_q}{T} = \mathbf{j}_q \cdot \nabla \frac{1}{T} - \frac{1}{T} \sum_{\nu=\uparrow,\downarrow} \mathbf{j}_\nu \cdot \nabla \bar{\mu}_\nu + \frac{1}{T} \sum_{\nu=\uparrow,\downarrow} \bar{\mu}_\nu \varphi_\nu. \quad (9)$$

The right-hand side of Eq. (9) is the entropy production rate²⁴ including the spin degree of freedom. The last term of Eq. (9) appears from the spin-dependent relaxation rate and has not appeared in the classical theory of dissipation, which does not take the spin current into account²⁵. Note that the derivation of Eq. (9) is independent of the explicit form of the particle current density. Therefore, Eq. (9) is general and valid for any type of spin current source (driving force), as emphasized earlier.

Tulapurkar and Suzuki¹ derived a different form of the entropy production rate formula in terms of the electrochemical potential $\bar{\mu} = (\bar{\mu}_\uparrow + \bar{\mu}_\downarrow)/2$ and the spin accumulation $\delta\mu = (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)/2$ by using the Boltzmann equation. Here, we show that the result in Ref.¹ is reproduced from Eq. (9) by applying the relaxation time approximation²⁶ to the spin relaxation rate given by

$$\varphi_\nu = \frac{n_\nu}{2\tau_{\text{sf}}^\nu} - \frac{n_{-\nu}}{2\tau_{\text{sf}}^{-\nu}}, \quad (10)$$

where τ_{sf}^ν is the spin-flip relaxation time from the spin- ν state to the opposite spin state. The nonequilibrium particle density, n_ν , is related to the density of state \mathcal{N}_ν

at the Fermi level ε_F via $n_\nu = \mathcal{N}_\nu(\mu_\nu - \varepsilon_F)$. The detailed balance, $\mathcal{N}_\nu/\tau_{\text{sf}}^\nu = \mathcal{N}_{-\nu}/\tau_{\text{sf}}^{-\nu}$, is satisfied in the steady state²⁷. Using Eq. (10) and these relations, we find that

$$\sum_{\nu=\uparrow,\downarrow} \bar{\mu}_\nu \varphi_\nu = \frac{(1 - \beta^2)}{e^2 \rho \ell^2} (\delta\mu)^2. \quad (11)$$

The spin diffusion length ℓ is defined as

$$\begin{aligned} \frac{1}{\ell^2} &= \frac{e^2}{2} \left(\frac{\mathcal{N}_\uparrow}{\sigma_\uparrow \tau_{\text{sf}}^\uparrow} + \frac{\mathcal{N}_\downarrow}{\sigma_\downarrow \tau_{\text{sf}}^\downarrow} \right) \\ &= \frac{1}{2} \left(\frac{1}{D_\uparrow \tau_{\text{sf}}^\uparrow} + \frac{1}{D_\downarrow \tau_{\text{sf}}^\downarrow} \right), \end{aligned} \quad (12)$$

where the diffusion coefficient D_ν is related to the conductivity of the spin- ν electrons, σ_ν , by the Einstein relation, $\sigma_\nu = e^2 D_\nu \mathcal{N}_\nu$. We also find from Eqs. (1) and (10) that

$$\nabla \cdot (\mathbf{j}_\uparrow - \mathbf{j}_\downarrow) = -\frac{(1 - \beta^2)\sigma}{e^2 \ell^2} \delta\mu. \quad (13)$$

Substituting Eq. (13) into (11), we obtain

$$\sum_{\nu=\uparrow,\downarrow} \bar{\mu}_\nu \varphi_\nu = -[\nabla \cdot (\mathbf{j}_\uparrow - \mathbf{j}_\downarrow)] \delta\mu. \quad (14)$$

Substituting Eq. (14) into Eq. (9) and using the conservation law of the particle current, $\nabla \cdot (\mathbf{j}_\uparrow + \mathbf{j}_\downarrow) = 0$, we find that

$$\frac{\partial \mathfrak{S}}{\partial t} + \nabla \cdot \frac{\mathbf{j}_q}{T} = \mathbf{j}_q \cdot \nabla \frac{1}{T} - \frac{1}{T} \nabla \cdot [(\mathbf{j}_\uparrow + \mathbf{j}_\downarrow) \bar{\mu}] - \frac{1}{T} \nabla \cdot [(\mathbf{j}_\uparrow - \mathbf{j}_\downarrow) \delta\mu]. \quad (15)$$

The second and third terms on the right-hand side of Eq. (15) include the electric and spin current densities, $\mathbf{J}_c = -e(\mathbf{j}_\uparrow + \mathbf{j}_\downarrow)$ and $\mathbf{J}_s = (\hbar/2)(\mathbf{j}_\uparrow - \mathbf{j}_\downarrow)$, respectively. Equation (15) is identical to the dissipation formula in Ref.¹ derived using the Boltzmann equation. We denote the right hand side of Eq. (15) as Σ_V for convenience, that is,

$$\Sigma_V = \mathbf{j}_q \cdot \nabla \frac{1}{T} - \frac{1}{T} \nabla \cdot [(\mathbf{j}_\uparrow + \mathbf{j}_\downarrow) \bar{\mu}] - \frac{1}{T} \nabla \cdot [(\mathbf{j}_\uparrow - \mathbf{j}_\downarrow) \delta\mu]. \quad (16)$$

We emphasize that Σ_V is the bulk entropy production rate. Similarly, the interface entropy production rate is given by¹

$$\Sigma_A = \hat{\mathbf{n}} \cdot \mathbf{j}_q \Delta \frac{1}{T} - \frac{1}{T} \hat{\mathbf{n}} \cdot [(\mathbf{j}_\uparrow + \mathbf{j}_\downarrow) \Delta \bar{\mu}] - \frac{1}{T} \hat{\mathbf{n}} \cdot [(\mathbf{j}_\uparrow - \mathbf{j}_\downarrow) \Delta \delta\mu], \quad (17)$$

where $\hat{\mathbf{n}}$ is the unit vector normal to the interface, and $\Delta(1/T)$, $\Delta\bar{\mu}$, and $\Delta\delta\mu$ are the differences in $1/T$, $\bar{\mu}$, and $\delta\mu$ at the interface, respectively. We assume that the heat, electric, and spin currents are continuous at the interface. The total entropy production rate is given by the sum of Eqs. (16) and (17) integrated over the volume and interfaces, respectively. For example, let us

consider the total entropy production rate in the current-perpendicular-to-plane spin valve consisting of two ferromagnets, F_1 and F_2 , and a nonmagnet N . Assuming that the electric current flows along the x direction, and summing the bulk entropy production rates Σ_V in the F_1 , N , and F_2 layers and the interface entropy production rates Σ_A at the F_1/N and N/F_2 interfaces, the total entropy production per unit area per unit time becomes

$$\begin{aligned} & \int_{-\infty}^{F_1/N} dx \Sigma_V + \Sigma_A(F_1/N) + \int_N dx \Sigma_V + \Sigma_A(N/F_2) + \int_{N/F_2}^{\infty} dx \Sigma_V \\ &= \frac{J_c}{eT} [\bar{\mu}(\infty) - \bar{\mu}(-\infty)], \end{aligned} \quad (18)$$

as derived in Ref.¹, where the system is connected to the electrodes at $x \rightarrow \pm\infty$. We assume that $\lim_{x \rightarrow \pm\infty} \delta\mu = 0$ and the temperature profile is uniform. Note that $\bar{\mu}(\infty) - \bar{\mu}(-\infty)$ in Eq. (18) is the potential difference between the electrodes, which drives the electric current. The entropy production rate originating from the spin Hall effect is calculated in a similar way, as shown below. Similarly, when the system is connected to a heat bath, the entropy production rate will be related to the temperature difference between the boundaries, as implied by the first terms of Eqs. (16) and (17). Another example of the previously reported entropy production rate is that in spin pumping⁴. In this case, a ferromagnetic(F)/nonmagnetic(N) interface with the ferromagnet in resonance plays a role similar to the electrodes in the spin valve because the spin current is driven from this interface. Then, as the result of an integral similar to that in Eq. (18), the total entropy production rate per unit area becomes $[J_s/(\hbar T/2)](\delta\mu_N - \delta\mu_F)$ as studied in Ref.⁴, where J_s , $\delta\mu_N$, and $\delta\mu_F$ are the total spin current pumped at the F/N interface and the spin accumulations at the interfaces of the N and F layers, respectively. We note that J_s and $\delta\mu_N - \delta\mu_F$ in this expression correspond to J_c and $\bar{\mu}(\infty) - \bar{\mu}(-\infty)$, respectively, in Eq. (18). In both cases, the total entropy production rate per is given by the current multiplied by the energy (potential difference) driving this current, as expected from thermodynamics²⁴.

Now let us apply the above formula to estimate the dissipation due to the spin current by considering the spin Hall system as an example. The system under consideration is shown in Fig. 1, where an electric field, E_x , applied to a nonmagnet along the x direction drives the spin-dependent electric current $\mathbf{J}_{c,\nu}$. The length along the x direction, the width along the y direction, and the thickness along the z direction are denoted as L , w , and d , respectively. We assume a homogeneous temperature profile, that is, $\nabla T = \mathbf{0}$, for simplicity. The spin-orbit interaction in the nonmagnet scatters the electrons in the transverse direction, where the scattering direction is spin-dependent. In the spin Hall geometry, the electric current densities carried by the spin-up and spin-down

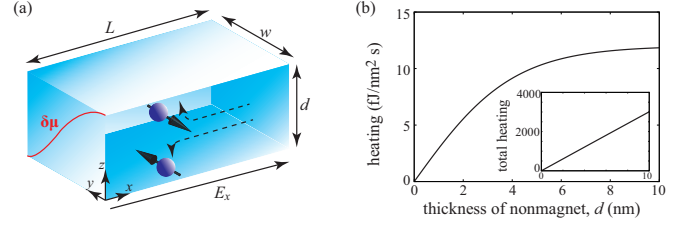


FIG. 1: (a) Schematic view of the spin Hall system. (b) Dependence of the second term of Eq. (25), divided by the cross-sectional area Lw , on the thickness of the nonmagnet d . Inset shows the total dissipation [sum of the first and second terms of Eq. (25)].

electrons are given by

$$\mathbf{J}_{c,\uparrow} = \frac{\sigma_N}{2e} \nabla \bar{\mu}_{\uparrow} + \frac{\vartheta \sigma_N}{2e} \hat{\mathbf{s}} \times \nabla \bar{\mu}_{\uparrow}, \quad (19)$$

$$\mathbf{J}_{c,\downarrow} = \frac{\sigma_N}{2e} \nabla \bar{\mu}_{\downarrow} - \frac{\vartheta \sigma_N}{2e} \hat{\mathbf{s}} \times \nabla \bar{\mu}_{\downarrow}, \quad (20)$$

where σ_N , ϑ , and $\hat{\mathbf{s}}$ are the conductivity of the nonmagnet, the spin Hall angle, and the unit vector pointing in the direction of the spin polarization, respectively. The electric current density and spin current density are

$$\mathbf{J}_c = \frac{\sigma_N}{e} \nabla \bar{\mu} + \frac{\vartheta \sigma_N}{e} \hat{\mathbf{s}} \times \nabla \delta\mu, \quad (21)$$

$$\mathbf{J}_s = -\frac{\hbar \sigma_N}{2e^2} \nabla \delta\mu - \frac{\hbar \vartheta \sigma_N}{2e^2} \hat{\mathbf{s}} \times \nabla \bar{\mu}. \quad (22)$$

It was shown²⁸ that the relaxation time approximation is applicable to the diffusion equation of the spin accumulation $\delta\mu$. Thus, Eq. (15) can be used to derive the entropy production rate formula in the spin Hall geometry.

We assume that the width w is sufficiently large, and the spatial variations of $\bar{\mu}$ and $\delta\mu$ along the y direction are negligible, as in the case of the experiments. Then, the spatial derivatives of $\bar{\mu}$ and $\delta\mu$ are given by $\nabla(\bar{\mu}/e) = E_x \mathbf{e}_x + \partial_z(\bar{\mu}/e) \mathbf{e}_z$ and $\nabla(\delta\mu/e) = \partial_z(\delta\mu/e) \mathbf{e}_z$, respectively. Applying the open circuit conditions of the electric current along the z direction, we find that $\partial_z \bar{\mu} = 0$. Thus, the electrochemical potential is $\bar{\mu} = eE_x x$. On the other hand, by solving the diffusion equation of the spin accumulation $\delta\mu$ with the open circuit conditions of the spin current and Eq. (22), the solution of $\delta\mu$ is given by

$$\delta\mu = \frac{-e\vartheta E_x \ell}{\sinh(d/\ell)} \left[\cosh\left(\frac{z-d}{\ell}\right) - \cosh\left(\frac{z}{\ell}\right) \right], \quad (23)$$

where ℓ is the spin diffusion length of the nonmagnet. Note that the spin accumulation is related to the electric current along the x direction as $\mathbf{e}_x \cdot \mathbf{J}_c = \sigma_N E_x + (\vartheta \sigma_N / e) \partial_z \delta\mu$. Let us denote Eq. (16) integrated over the volume of the nonmagnet and multiplied by the temperature as $\partial Q_V / \partial t = \int dx dy dz T \Sigma_V$, which is the dissipation (heating) per unit time in this system. Using the above

result, $\nabla(\bar{\mu}/e) = E_x \mathbf{e}_x$, and applying the open-circuit condition to the spin current, we find that

$$\frac{\partial \mathcal{Q}_V}{\partial t} = \int dxdydz \left(\sigma_N E_x + \frac{\vartheta \sigma_N}{e} \partial_z \delta \mu \right) E_x. \quad (24)$$

Substituting Eq. (23) into Eq. (24), we find that

$$\frac{\partial \mathcal{Q}_V}{\partial t} = \sigma_N E_x^2 Lwd + 2\vartheta^2 \sigma_N E_x^2 Lw\ell \tanh\left(\frac{d}{2\ell}\right). \quad (25)$$

The first term of Eq. (25) is the conventional Joule heating, which is the product of the electric current density $\sigma_N E_x$ and the electric field E_x integrated over the volume. On the other hand, the second term represents the contribution of the inverse spin Hall effect to the dissipation. This term is proportional to the square of the spin Hall angle and becomes saturated for a thickness sufficiently thicker than the spin diffusion length. Figure 1(b) shows an example of the dependence of the second term of Eq. (25) on the thickness of the nonmagnet, d . In the figure, the second term of Eq. (25), shown on the vertical axis, is divided by the cross-sectional area in the xy plane (Lw), that is, $2\vartheta^2 \sigma_N E_x^2 \ell \tanh[d/(2\ell)]$. The values of the parameters are $\rho_N = 1/\sigma_N = 3000 \text{ } \Omega\text{nm}$, $\ell = 2 \text{ nm}$, and $|\vartheta| = 0.1$, which are typical values found in experiments for nonmagnetic heavy metals^{14,15,29,30}. The electric field is $E_x = J_{c0}/\sigma_N$, where the current density

is $J_{c0} = 10^6 \text{ A/cm}^2$. As shown, this dissipation is on the order of $10 \text{ fJ/(nm}^2\text{s)}$, which is comparable to that found in the other system⁴. Because the spin Hall angle is small, this dissipation is much smaller than the total dissipation, which is shown in the inset of Fig. 1(b).

In conclusion, we developed a comprehensive theory of dissipation due to a spin current by using the continuous equation of the spin current and thermodynamics. A formula for the entropy production rate that is applicable regardless of the spin current was derived. The previous work by Tulapurkar and Suzuki¹ is reproduced when the relaxation time approximation for the spin relaxation is applied to the present theory. We also found an additional term proportional to the square of the spin Hall angle contributing to the conventional Joule heating in the spin Hall geometry.

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